Comment on "Nucleon elastic form factors and local duality"

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We comment on the papers "Nucleon elastic form factors and local duality" [Phys. Rev. D **62**, 073008 (2000)] and "Experimental verification of quark-hadron duality" [Phys. Rev. Lett. **85**, 1186 (2000)]. Our main comment is that the reconstruction of the proton magnetic form factor, claimed to be obtained from the inelastic scaling curve thanks to parton-hadron local duality, is affected by an artifact.

Recently an inclusive electron-proton scattering experiment [1] has been performed at the Jefferson Lab (JLab) in the resonance production region for values of the squared four-momentum transfer Q^2 between ~ 0.45 and ~ 3.3 $(\text{GeV}/c)^2$. The aim was to investigate the connection between the resonance and the scaling regions, known as the parton-hadron local duality [2]. The new data were found to exhibit the local duality for each of the most prominent proton resonances. In Ref. [1] a fit to the average strength of all the resonances was carried out and, thanks to the parton-hadron local duality, interpreted as the *scaling* curve. Here below, we refer to such a fit as the JLab fit, viz.,

$$F_2^{(\text{JLab})}(\xi) = \xi^{0.870}(1-\xi)^{0.006}[0.005 - 0.058(1-\xi) - 0.017$$
$$\times (1-\xi)^2 + 2.469(1-\xi)^3 - 0.240(1-\xi)^4],$$
$$(1)$$

where $\xi \equiv 2x/[1+\sqrt{1+4M^2x^2/Q^2}]$ is the Nachtmann variable, which includes the effects of target-mass corrections, improving at finite Q^2 the Bjorken scaling variable x. In order to constrain the large-ξ behavior of the JLab fit the authors of Ref. [1] have employed SLAC data up to $Q^2 = 8$ $(\text{GeV}/c)^2$. This means that the highest ξ-point constraining the JLab fit is $\xi \simeq 0.86$, corresponding to the $\Delta(1232)$ location at $Q^2 = 8$ $(\text{GeV}/c)^2$.

An interesting question is whether local duality may be applied to the proton elastic peak [2–5]. If local duality holds also in the unphysical region extending up to $\xi = 1$ (which corresponds at finite Q^2 to $x > 1$), the proton magnetic form factor $G_M^p(Q^2)$ can be obtained from the moment of order n of the scaling function, $F_2^p(\xi)$, viz. (see Ref. [3])

$$G_M^p(Q^2) = \sqrt{\frac{2-\xi_{el}}{\xi_{el}^n}\mu_p^2 \frac{1+\tau}{1+\mu_p^2\tau}\int_{\xi_\pi}^{\xi^*} d\xi \xi^{n-2} F_2^p(\xi)}, \quad (2)$$

where μ_p is the proton magnetic moment, $\tau = Q^2/4M^2$, $\xi_{el} = 2/[1+\sqrt{1+1/\tau}]$, $\xi^* = \min[1, Q/M]$ and ξ_π is the pion production threshold. Note that $\xi_\pi(\xi_{el}) = 0.41(0.50)$, $0.63(0.70)$, $0.76(0.81)$, $0.83(0.87)$, $0.90(0.92)$ at $Q^2 = 0.45$, 1.4, 3.0, 5.0, 10 $(\text{GeV}/c)^2$, respectively. In Ref. [6], adopting for $F_2^p(\xi)$ the JLab fit (1) and considering only $n = 2$, the reconstructed $G_M^p(Q^2)$

was shown to agree with the data within 30% up to $Q^2 \sim 7$ $(\text{GeV}/c)^2$. This result is at variance with the findings of Refs. [2–5].

We start noting that in the right-hand side of Eq. (1) the term proportional to $(1-\xi)$, which any way is not consistent with quark counting rules, has a negative coefficient, so that $F_2^{(\text{JLab})}(\xi)$ may be a monotonic increasing function at large ξ. This is indeed the case as shown in Fig. 1 by the dashed line,[1] which exhibits an anomalous behavior at $\xi \gtrsim 0.9$, i.e., beyond the highest ξ point constraining the JLab fit of Ref. [1].

In order to clarify the impact of the anomalous shape of the JLab fit on the reconstruction of $G_M^p(Q^2)$ through Eq. (2), we have simply developed a *modified* version of the JLab fit, which coincides with the original one within $\pm 10\%$

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[1]Note that in Eq. (1) the term $(1-\xi)^{0.006}$ ensures that $F_2^{(\text{JLab})}(\xi = 1) = 0$, but in practice it has no effect at all for ξ up to 0.9999, as it can be easily checked numerically.
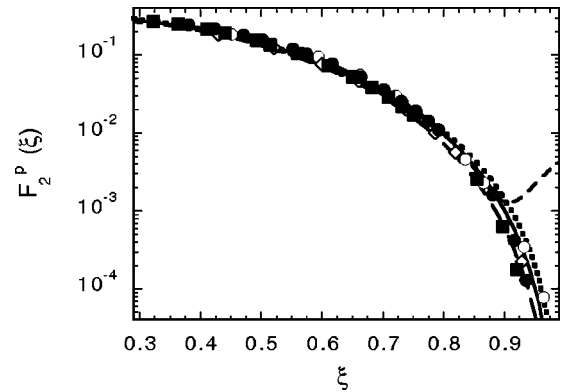

FIG. 1. The proton scaling function $F_2^p(\xi)$ versus the Nachtmann variable ξ. The dashed and solid lines correspond to the JLab fit (1) of Ref. [1] and to our modified JLab fit given by Eq. (3), respectively. The open dots and diamonds, and the full dots and squares are the average strengths, obtained using the SLAC parametrization [7] of the proton structure function, in the $\Delta(1232)$, $S_{11}(1535)$, $F_{15}(1680)$ and "higher-mass" resonance regions, as defined in Ref. [1], respectively. The long-dashed and dotted lines correspond respectively to the GRV set [8] of PDF's and to the NMC parametrization [9], omitting for the latter its $1/Q^2$ term (see text), evaluated at $Q^2 = 10$ $(\text{GeV}/c)^2$.

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for $\xi \lesssim 0.86$, but exhibits a monotonic decreasing behavior at larger ξ, viz.,

$$\tilde{F}_2^{(\text{JLab})}(\xi) = \xi^{0.940}[2.650(1-\xi)^{3.38} + 0.240(1-\xi)^4]. \quad (3)$$

In Fig. 1 the modified JLab fit is reported as the solid line and compared with the average strengths of the most prominent proton resonances, generated using the parametrization of the inelastic SLAC data of Ref. [7]. It can be seen that our modified JLab fit is in reasonable agreement with the SLAC resonance averages up to very large values of ξ. Finally, the proton structure function $F_2^p(\xi)$ evaluated at $Q^2 = 10$ $(\text{GeV}/c)^2$ using the GRV set [8] of parton distribution functions (PDF's) and the NMC parametrization of Ref. [9] is shown in Fig. 1. Note that the NMC fit contains an explicit power correction term proportional to $1/Q^2$, which has been excluded. Indeed, as shown in Ref. [3], the replacement of the Bjorken variable x with the Nachtmann variable ξ is an approximate way to consider target mass (TM) corrections at large Q^2. Therefore, the inclusion of the $1/Q^2$ term of the NMC fit (which incorporates already TM effects) and, at the same time, the use of the variable ξ would lead to an overcounting of TM effects. From Fig. 1 it can be seen that up to $\xi \simeq 0.85$ the shape of the Jlab fit is not inconsistent with standard PDF expectations as well as with the NMC fit, provided TM effects are properly included. This is at variance with the results reported in Fig. 2 of Ref. [1]. There, however, TM effects were not included consistently in the curves labeled "MRS(G)" and "CTEQ4." We have checked that, after proper inclusion of TM corrections, those curves become close to the results labeled "NMC" (see also Fig. 3 of [3]).

In Fig. 2 the proton magnetic form factor $G_M^p(Q^2)$, resulting from the application of the parton-hadron local duality [Eq. (2)] using the original and our modified JLab fits, is shown and compared with the data. In evaluating Eq. (2) we have considered both $n=2$ and $n=10$. The former case is the only one employed in Ref. [6], while the latter is representative of the case of higher moments which are more sensitive to the shape of the scaling curve at large ξ. It can be seen that (i) the results we have obtained using the JLab fit
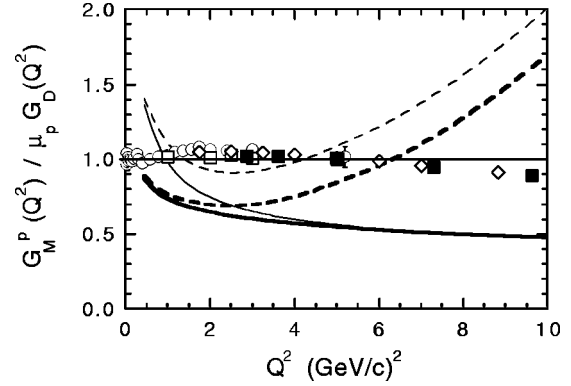

FIG. 2. The proton magnetic form factor $G_M^p(Q^2)$, divided by its dipole expectation $\mu_p G_D(Q^2) \equiv 2.793/(1+Q^2/0.71)^2$, versus Q^2. Open dots, squares, diamonds, and full squares are the experimental data from Ref. [10] (a), (b), (c), and (d), respectively. The dashed and solid lines are the results of Eq. (2) obtained using the JLab fit (1) and its modified version (3), respectively. Thick and thin lines correspond in Eq. (2) to $n=2$ and $n=10$, respectively.

(1) coincide with those of Ref. [6] for $n=2$ (thick dashed line), but exhibit a remarkable dependence on the order n of the moment (compare thin and thick dashed lines); (ii) the anomalous shape of the JLab fit (1) heavily affects the reconstruction of $G_M^p(Q^2)$ for $Q^2 \gtrsim 5$ $(\text{GeV}/c)^2$ (compare thick dashed and solid lines); (iii) using our modified JLab fit the values of $G_M^p(Q^2)$ obtained via the application of parton-hadron local duality, underestimates the data by a factor of $\simeq 2$ for $Q^2 \gtrsim 2$ $(\text{GeV}/c)^2$ (see thick solid line); (iv) for $Q^2 \lesssim 2$ $(\text{GeV}/c)^2$ the reconstructed $G_M^p(Q^2)$ appears to be close to the experimental data only if $n=2$ is adopted (compare thin and thick lines).

To sum up, the main conclusion of Ref. [6], concerning the possibility of reconstructing the proton magnetic form factor from the inelastic scaling curve, is the result of an artifact in the JLab fit (1) of Ref. [1]. Using our modified JLab fit [see Eq. (3)] we have shown that the application of the parton-hadron local duality, as given by Eq. (2), fails to reproduce existing data on $G_M^p(Q^2)$ at least for Q^2 up to ~ 10 $(\text{GeV}/c)^2$, in agreement with the findings of Refs. [2–5].

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